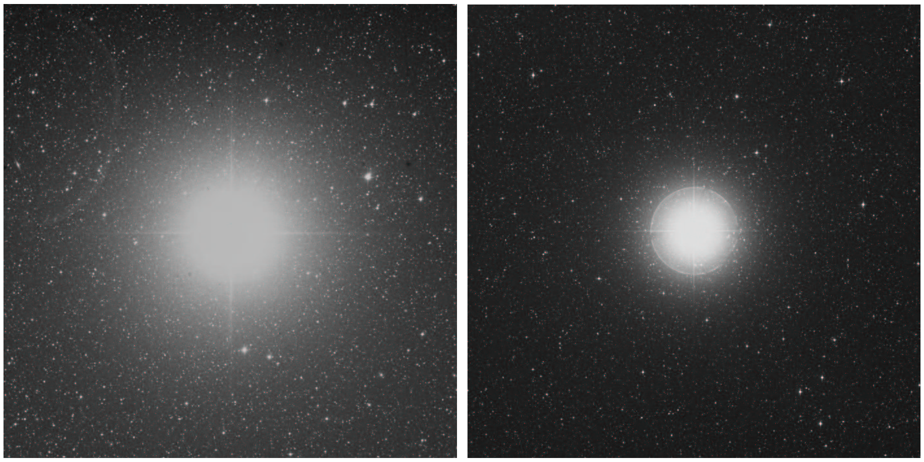
Study of a $3\mathscr{M}_\odot$ Star

prepared for Dr. Jim Sowell's Stellar Astrophysics class

David T. Eger <eger@cc.gatech.edu>
James Giessner <gte717n@prism.gatech.edu>
Luke Rosshirt <gtg247a@prism.gatech.edu>
Georgia Institute of Technology



Vega [left], a $2.5\mathscr{M}_\odot$ star, is the fifth-brightest star in the night sky. Rukbat [right], a $3.2\mathscr{M}_\odot$ star in the constellation Sagittarius, is a faint star in the southern hemisphere. Both stars have similar characteristics to the hypothesized 3 $\mathscr{M}_\odot$ star we have derived and describe in this paper.

Images are 60' x 60' and provided courtesy the Space Telescope Science Institute's Digitized Sky Survey Project

The Lives of Stars

Through the centuries, men have looked up at the night sky to admire the tiny shining lights. In 1900, how the stars shone was anyone's guess. No one knew anything about an atom's interior. But people did know about fire. If you heat wood high enough and give it air, it will burn. This process was understood - a molecular reaction releasing carbon dioxide and water. During this past century, scientists have learned that analogously, at high enough pressures and temperatures, atomic nuclei will react to form different atoms. Hydrogen atoms, for example, will combine to form helium, releasing a great deal more energy than the molecular combination of two hydrogen atoms to form hydrogen gas. Such nuclear processes are what provide the energy with which stars shine.


Hubble-X in NGC 6822

NASA and The Hubble Heritage Team (STScI/AURA)
Hubble Space Telescope WFPC2 • STScI-PRC01-01

Figure 1: The Hubble-X Nebula (giant gas cloud), a birthplace for stars.

One might wonder then on the fact that there is a lot of hydrogen in the Earth's oceans. Fortunately, though, the Earth is not going to start to burn away due to the fusion of hydrogen. For gravity to generate the kinds of pressure and temperatures needed to cause hydrogen to fuse, one needs to amass a quantity of hydrogen over 26,000 times the mass of the Earth! With less mass than that, the force of pressure wins out over the force of gravity. This is the eternal battle of the cosmos, pressure versus gravity.

In the diffuse gas that is the majority of the interstellar medium, simple pressure, low as it is in that near vacuum, is strong enough to fend off the inifinitesimaly smaller affect of gravity that might pull the gas particles together. However, when a large enough cloud of gas finds its way together, gravity wins, and the cloud will collapse.

Just as different chemical reactions take place with different reactants, different nuclear reactions take place with gaseous clouds of different compositions. So the sort of star that forms as a cloud condenses into a fiery ball is dependent upon what sort of gas is condensing. One of the landmark statements that has come from the study of the stars is that it is from the atomic composition of a gas cloud, and its size, that the sort of star that is formed is completely determined. This is known as the Vogt-Russell theorem.

Hydrogen is estimated to compose 70% of the mass in the universe and is the easiest thing to fuse. The temperatures and pressures required for hydrogen fusion are much lower than those required for the fusion of other elements, like carbon or oxygen. Therefore, stars begin their lives burning hydrogen, and in fact spend the majority of their lives doing just that. Stars of different compositions and masses will live different kinds of lives - and may have spectacularly different deaths. These differences are what make this project so interesting. In this paper we explore what a day in the hydrogen-burning life of a very particular star would be. We are given the star's mass and composition, and we are challenged to find a model conforming to the laws which govern the universe of what must be going on inside.

Our Task

We examine the life of a star which is 70% hydrogen, 29.2% helium, and 0.8% other elements by mass, and which weighs in at 3.0 times the mass of the Sun (written 3 $\mathscr{M}_\odot$). We are also given the effective Temperature at the surface of the star, $T_e = 15007.4K$ to help us get started.

To Model a Star

To build a model for our star, we simply need to determine how much energy ($L_\star$=luminosity) such a star would produce. With that, we can use the relation $L = A\sigma {T_e}^4 = 4\pi R_\star^2 \sigma {T_e}^4$ to find the radius of our star.

Once we have the radius, we use the laws of nature to show us what must be happening inside. First, we assume that the amount of mass in the star remains constant - that is, that a negligible amount is lost by drifting into space or being converted to energy. One might question this assumption - our own 1.0 $\mathscr{M}_\odot$ sun is burning hydrogen, and is losing $1.34 \times 10^{20} kg/yr$ mass as energy by that process. This amount, though is on the order of $1/10^{10}$ the mass of the sun, so perhaps our estimation is good enough to use to describe a moment - but perhaps not an entire stage - in the life of our star. We also assume a hydrostatic equilibrium, that is, that there is a steady pressure gradient in the star which keeps the outer layers from collapsing in. For a star that is not pulsating, and is shining bright and steady for as long as we have watched it, this seems a reasonable thing to assume.

Finally, we assume the laws of thermodynamics. We know that energy is carried by conduction, convection and radiation. Further, scientists have determined that which one is the main source of energy transport in a star depends on the gradients of pressure and luminosity in the area in question. However, details about the mass movements of stellar gases draw upon parts of fluid mechanics and thermodynamics that are not well understood. All is not lost, however, as we do have some approximations, as crude as they may be.

To glimpse at a day in the life of our 3.0 $\mathscr{M}_\odot$ we use a Fortran code called STATSTAR which takes all of the laws we have mentioned above into account. We enter the mass of our star, its effective temperature, and a guess at what luminosity we think such a star might have, and the code simply integrates the differential equations that embody the laws of physics which we have just described. Starting at the surface of the star, where the pressure is minimal, the temperature is minimal, and the luminosity is full, the program, shell by shell, estimates the mass, pressure, temperature, and luminosity that must appear at the shell beneath, going all of the way to the core.

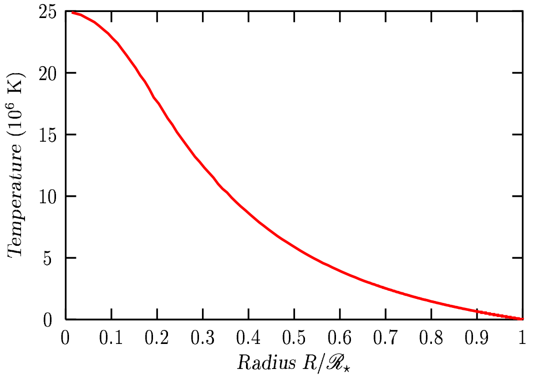
In most cases, our guesses will not work. The Vogt-Russell Theorem says that we should be able to find only one set of values that does work for our mass and composition. If we make a guess of luminosity that is too high, it the simulation will run out of mass before it reaches the center of the star. Or perhaps we guess too low, and 3 $M_\odot$ is more mass than we need. In either case, the code will eject an error message instead of a good model of our star. It is by trying to determine when the error messages that STATSTAR produces change that our group honed in on a luminosity that worked, $L_\star = 116.58 L_\odot$.

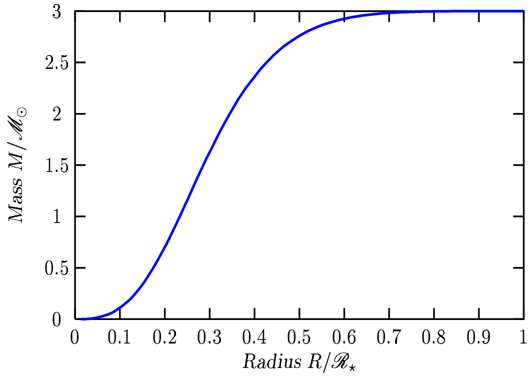
Description of a 3.0 $\mathscr{M}_\odot$ Star

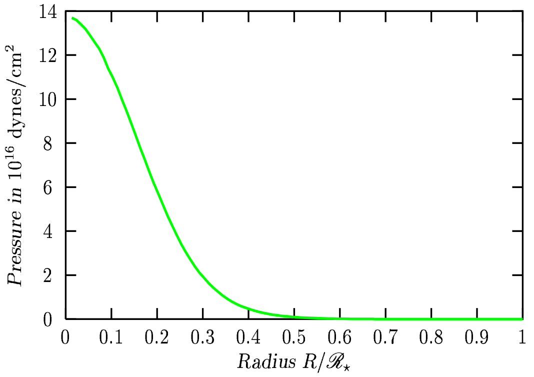
We found that for our 3.0 $\mathscr{M}_\odot$ star, $L_\star = 116.58 L_\odot$, $R_\star = 1.596 R_\odot$. 50% of the energy production occurs in a tiny core of radius $R_{50\%L_\star} = 0.141 R_\odot$, generating energy out of a mere $M_{50\%L_\star} = 0.077 \mathscr{M}_\odot$. This means that a tiny portion of the star – 2.6% of its mass residing in .069% of its volume – produces the half of the star's energy!

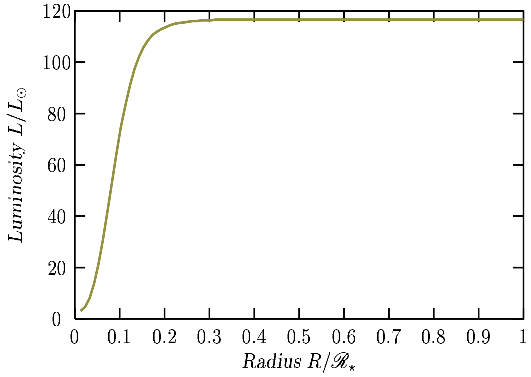
Virtually all of the energy (99%) that such a star produces is produced within $R_{99\%L_\star} = 0.388 R_\odot = 24\% R_\star$ with $M_{50\%L_\star} = 1.10 \mathscr{M}_\odot = 37\% M_\star$.

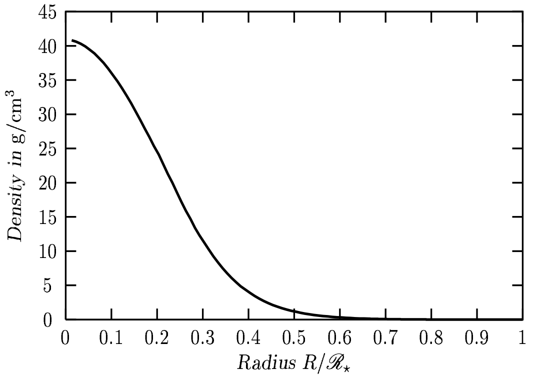
As shown in the data generated by STATSTAR, these patterns are expected, since it is in the center of the star where the rest of the star is pushing down that there is enough temperature and pressure for the fusion of hydrogen to occur.

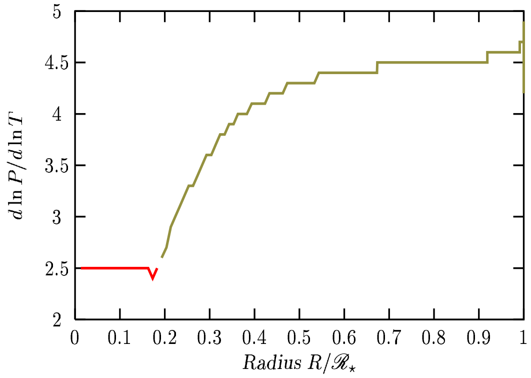






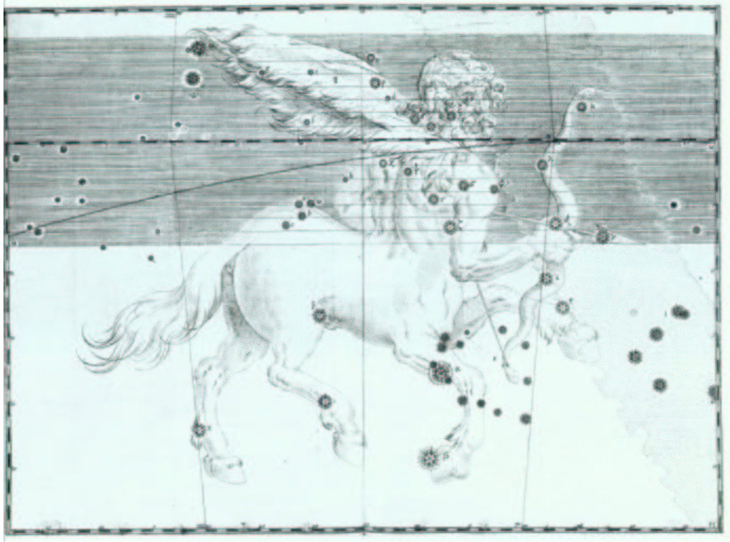






Stars Like Ours

So how does our star compare to other stars? Our Sun, which is a third as massive, is also much cooler. Where our 3 $\mathscr{M}_\odot$ star reaches twenty-five million degrees at its core, the Sun reaches only about fifteen million degrees. This is good for us, since the more massive stars burn out more quickly. A 3 $\mathscr{M}_\odot$ star burns out in about 250 million years, which wouldn't have given us time to have evolved. Also, the thermodynamics of the stars are different. The chart above of $d\ln P/d\ln T$ shows an interesting characteristic. One of the qualities that physicists have determined is that if this quantity is below 2.5, a star tends to move energy by convection, otherwise, the star uses radiation. So our $\mathscr{M}_\odot$ star has a convective core, but is mostly radiative. The Sun is just the opposite. It has a radiative core, but is convective further out.



A drawing of Sagittarius from
Bayer's "Uranometria" of 1602

The most famous stars that are closest in characteristics to our 3 $\mathscr{M}_\odot$ star are Vega and Rukbat (Alpha Sagittarius), which are shown on the cover of this report. Vega is an A0V star, and Rukbat a B9V, meaning that Rukbat burns a tiny bit hotter than Vega and is a little bit more massive. Vega is much closer, though, and is one of the brightest stars in the night sky. Rukbat is 170 light years away, radiates 112 $L_\odot$ from its 12370 K surface, and has 2.3 times the radius of the Sun. Vega is 25 light years away, radiates 50 $L_\odot$ from its 9500 K surface, and has a radius about 2.7 times that of the Sun.